Assignment 3

***Due**: Monday, October 27th, 2025 @ 11:59PM*

CPSC 4470/5470 Introduction to Quantum Computing (Fall 2025)

1 Introduction

Welcome to Assignment 3 for CPSC 4470/5470 (Introduction to Quantum Computing). As usual, collaboration is encouraged; if you discussed with anyone besides the course staff about the assignment, *please list their names* in your submission.

Getting Started.

This assignment has *only one* part, the *written portion*. Typesetting your solutions to the written portion is not mandatory but highly encouraged. See the instructor's note on Ed for details about Latex for quantum computing. To start,

- Create a folder for Assignment 3, e.g., `A3/`
- Download the starter files for this assignment to that folder from the course website:
  - `A3.pdf`
  - `written.tex`
- Write your solutions in `written.tex`

There are now two types of tasks, *mandatory* or *optional*. *Only the mandatory tasks will be counted towards your final grade for this assignment.* Different from the mandatory tasks, the optional tasks will be marked with "**(⋆ pts)**".

Submission.

Once you have completed and are ready to submit, upload to Canvas: `written.pdf`. Late submissions (for up to two days) will receive a 15% penalty. Your written solution will be graded manually by our course staff.

WRITTEN PORTION
This portion of the assignment has a total of 100 points.
Any tasks marked with (⋆ pts) are optional.

2 Power of Entanglement

Task 2.1 (15 pts)

Superdense coding. Suppose Alice and Bob share prior entanglement between them. That is, they each holds on to one of the qubits in the Bell state: $|\psi_{AB}\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. Now, Alice wants to send *two (classical) bits* of information to Bob. We will show in this question that she can do that by sending only *one qubit*.

(a) Consider two classical bits $a, b \in \{0,1\}$. Alice encodes the two bits in her qubit following the steps below:

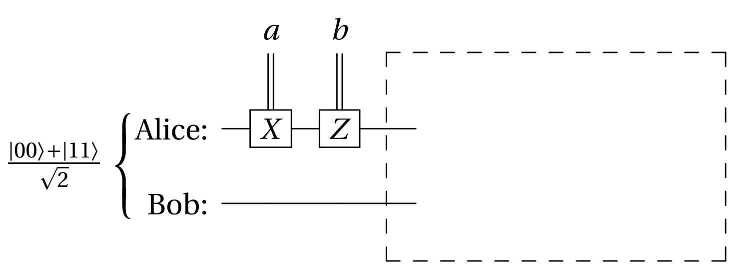
1. If $a = 1$, she performs Pauli X gate. Otherwise, she skips to Step 2.
2. If $b = 1$, she performs Pauli Z gate. Otherwise, she skips to Step 3.
3. She sends her qubit directly to Bob.

Show that Bob can fully determine Alice's bits a, b by measuring the qubits in an appropriate basis. (Hint: define a basis by a set of four orthonormal two-qubit states.)

(b) Provide the part of the quantum circuit that implements Bob's operations using only gates from

$$\{X, Y, Z, H, CNOT\}$$

and computational-basis (z-basis) measurements.



Bob's operations in dashed box

3 Quantum Oracles

Task 3.1 (10 pts)

Recall from lecture, we have defined the phase oracle for a function $f : \{0,1\}^n \to \{0,1\}$ as follows:

$$O_f^{\pm}|x\rangle = (-1)^{f(x)}|x\rangle.$$

Suppose we have an oracle to the following function $f : \{0,1\}^2 \to \{0,1\}$:

$$f(x) = \begin{cases} 1, \text{ if } x = 01 \\ 0, \text{ otherwise} \end{cases}$$

Given an input quantum state $|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |10\rangle)$. What is its state after applying the phase oracle to $|\psi\rangle$. That is, compute $O_f^{\pm}|\psi\rangle$.

Task 3.2 (10 pts)

What is the unitary matrix for the $O_f^{\pm}$ from the previous question? (Hint: $O_f^{\pm}$ can be viewed as a reflection operator.)

Task 3.3 (10 pts)

Give a circuit implementation of the $O_f^{\pm}$ from the previous questions.

Task 3.4 (20 pts)

Suppose now we are given an oracle to the following function $f : \{0,1\}^2 \to \{0,1\}$:

$$f(x) = \begin{cases} 1, \text{ if } x = 01 \text{ or } 00 \\ 0, \text{ otherwise} \end{cases}$$

Answer the previous three questions. That is, compute $O_f^{\pm}|\psi\rangle$, write down the unitary matrix for $O_f^{\pm}$, and give a quantum circuit for $O_f^{\pm}$.

4 Simon's Algorithm

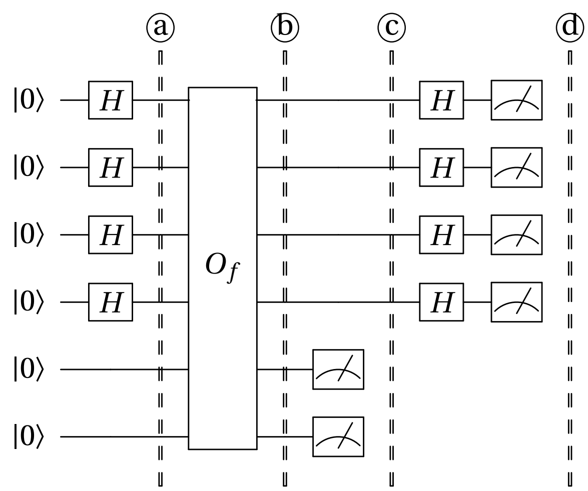
Task 4.1 (25 pts)

In Simon's algorithm, we are given a "periodic", two-to-one function and want to find its period. Let's analyze how this algorithm works for a particular function $f : \{0,1\}^3 \to \{0,1\}^2$, where $f(x \oplus a) = f(x)$ for some unknown period a. Importantly, here addition $\oplus$ is a *bit-wise addition modulo 2*. For example, $010 \oplus 011 = 001$. Its truth table looks like this in Table 1.

We have colored the output so that you can visually inspect the period. In Simon's algorithm, we use the following quantum circuit to find the period:

Input	Output		Input	Output
0000	$f(0000) = 00$		1000	$f(1000) = 00$
0001	$f(0001) = 01$		1001	$f(1001) = 01$
0010	$f(0010) = 00$		1010	$f(1010) = 00$
0011	$f(0011) = 01$		1011	$f(1011) = 01$
0100	$f(0100) = 10$		1100	$f(1100) = 10$
0101	$f(0101) = 11$		1101	$f(1101) = 11$
0110	$f(0110) = 10$		1110	$f(1110) = 10$
0111	$f(0111) = 11$		1111	$f(1111) = 11$

Table 1: Truth table of function f.



We have labeled four important time steps in the quantum circuit. Answer the following questions.

(a) Visually inspect the whole truth table in Table 1. What is the period a, such that $f(x \oplus a) = f(x)$ for all inputs x?

(b) What is the quantum state at time step ⓐ, i.e., after $H^{\otimes 4}$?

(c) What is the quantum state at time step ⓑ, i.e., after the oracle O_f?

(d) What is the quantum state at time step ⓒ, i.e., after measuring the bottom two qubits, given that the measurement outcome is 11? Please write your answer in the form of $(\alpha_{0000}|0000\rangle + \alpha_{0001}|0001\rangle + \cdots + \alpha_{1111}|1111\rangle) \otimes |11\rangle$ with the appropriate values for α_i. Here we assume top qubit in the circuit is the most-significant bit in the state, that is $|x\rangle = |x_3x_2x_1x_0\rangle$, $x = \sum_s x_s 2^s$, and $|x_3\rangle$ is the top qubit.

(e) What are the possible measurement outcomes at time step ⓓ, assuming the measurement outcome of 11 from (d)? Write down all possible outcomes and their associated probabilities.

Task 4.2 (10 pts)

Following the previous question,

1. If the measurement outcome at time step ⓓ is 0100, what do we know about the period a? In other words, what are the possible values of a that are consistent with this measurement outcome?

2. If we repeat Simon's algorithm and obtain another measurement outcome 1110 at time step ⓓ, what do we know about the period now? In other words, what are the possible values of a that are consistent with both the first measurement outcome 0100 and the second measurement outcome 1110?