Yale University

Coherent Data Loading

CPSC 4470/5470

Introduction to Quantum Computing

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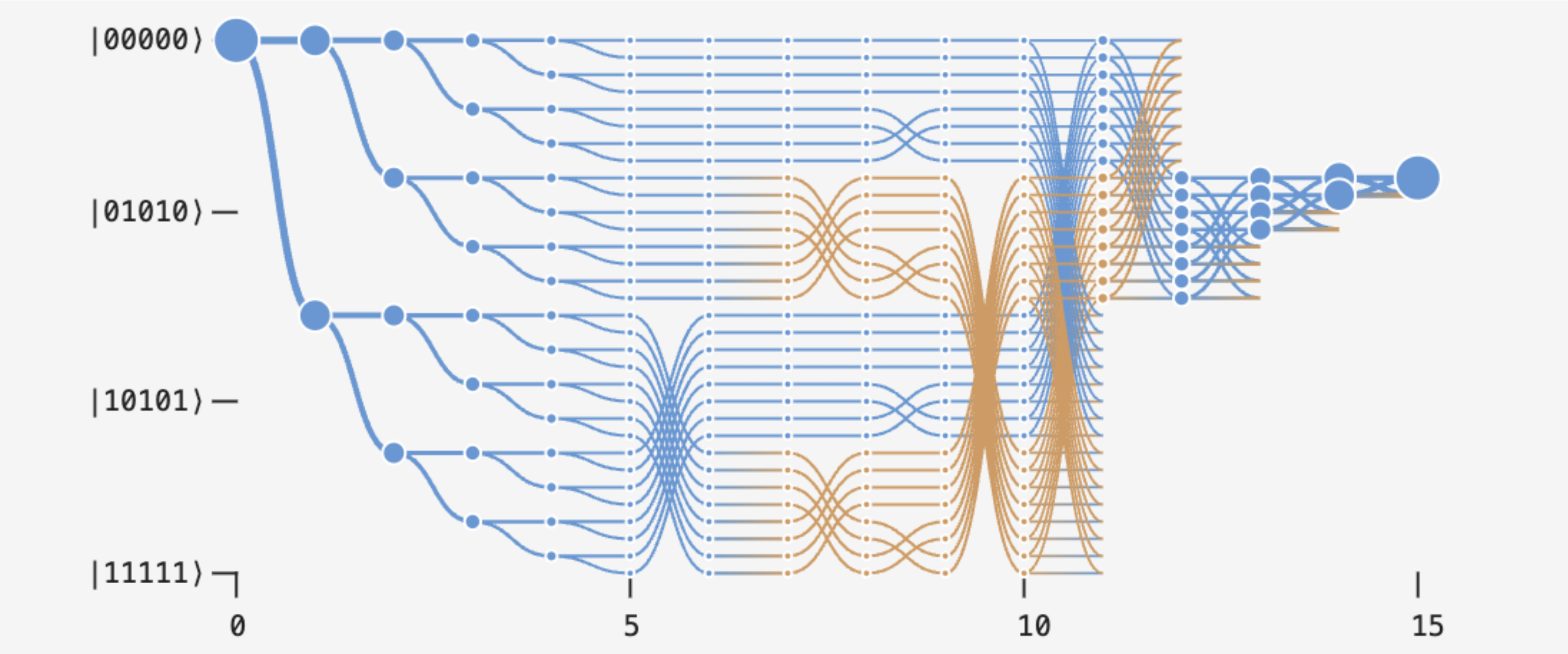
Massive Quantum Parallelism?

"Processing all data in superposition."



$$|\psi\rangle = \sum_{x \in \{0,1\}^n} \alpha_x \, |x\rangle$$

N-qubit state: superposition over 2^n computational basis states.

Data Input/Output for a Quantum Computer

Input (classical → quantum):

- How do we feed a **quantum computer** some **classical data** like words, images, sounds?

Output (quantum → classical):

- How much classical information can we extract from a quantum computer?

$$x, y \xrightarrow{\text{Embedding}} |\psi_x\rangle \xrightarrow[\text{Processing}]{U_y} U_y|\psi_x\rangle \xrightarrow{\text{Measurement}} z$$

Initial | Final

Classical Input Data | Quantum State | Quantum Circuit | Quantum State | Classical Output Data

Quantum Embeddings

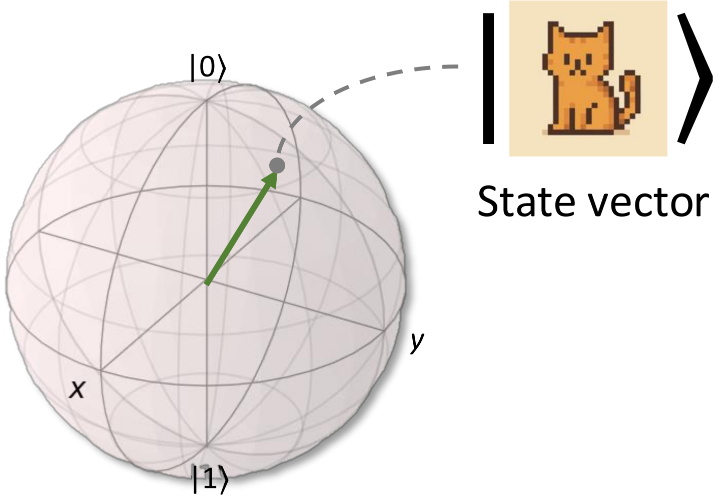
$x \rightarrow |\psi_x\rangle$

"A feature map from classical inputs to quantum states."

Words, images, sounds, …

[[71, 42, 27],
[73, 46, 32],
[74, 46, 33],
[74, 52, 44],
[132, 108, 82],
[219, 148, 79],
[241, 222, 188],
[243, 223, 188],
…]

Digital data



State vector

Quantum state

Yale

Quantum Embeddings

$x \rightarrow |\psi_x\rangle$ "A feature map from classical inputs to quantum states."

Example embeddings: for feature vector $x = (x_0, x_1, x_2)$ where x_i is a 32-bit floating-point number.

Digital data

x_0:

0101010…

x_1:

1101110…

x_2:

0001011…

Basis encoding:

$$|\psi_x\rangle = \frac{1}{\sqrt{3}}(|x_0\rangle + |x_1\rangle + |x_2\rangle)$$

(Used in some query algorithms)

Angle encoding:

$$|\psi_x\rangle = R_x(x_0) \otimes R_x(x_1) \otimes R_x(x_2)|000\rangle$$

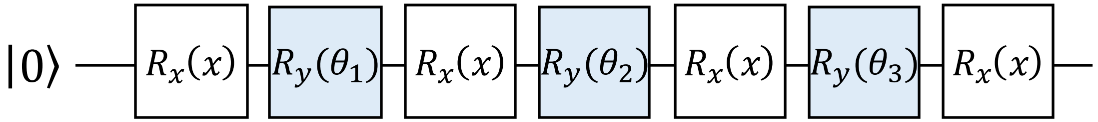
(Used in variational algorithms for QML)

Amplitude encoding:

$$|\psi_x\rangle = \frac{x_0|00\rangle + x_1|01\rangle + x_2|10\rangle}{\sqrt{x_1^2 + x_1^2 + x_1^2}}$$

(Used in HHL algorithm for linear systems)

Trainable encoding circuit:



Quantum Measurements

m-bit message **n-qubit state**

Alice x $\xrightarrow{\text{Embedding}}$ $|\psi_x\rangle$ $\xrightarrow{\text{Send qubit(s)}}$ $|\psi_x\rangle$ $\xrightarrow{\text{Measurement}}$ z Bob

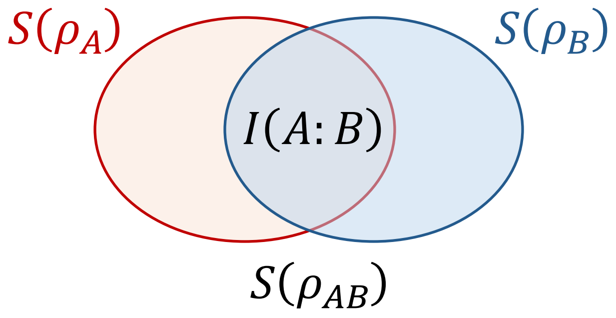
Classical Input Data | Quantum communication | Classical Output Data

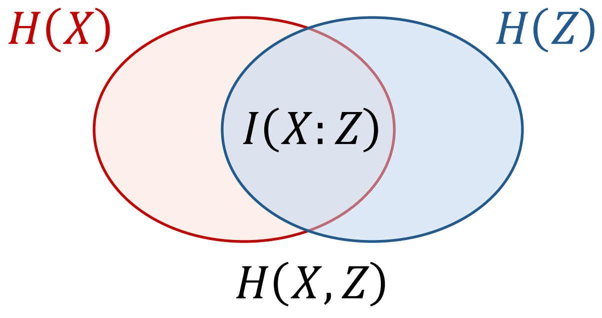
How many bits of information does Bob receive? Depends on the embeddings and measurements.

- **Holevo's theorem:**
  - If encoded in n qubits ($n < m$), Bob gets **at most $\boldsymbol{n}$ bits** of information about x.
  - No matter how clever the embedding or measurement strategies.

- **Example**: If Alice encodes x in 1 qubit ($n = 1$):
  - How many bits (of information about x) does Bob get?

Quantum Measurements





Random bits | **n-qubit state** | **Random bits**

Alice x $\xrightarrow{\text{Any embedding}}$ $|\psi_x\rangle$ $\xrightarrow{\text{Send qubit(s)}}$ $|\psi_x\rangle$ $\xrightarrow{\text{Any measurement}}$ z Bob

Ensemble: $\{p(x), x\}$

$$p(x), \rho_x \Rightarrow \sum_x p(x)\rho_x$$

Ensemble: $\{q(z), z\}$

How many bits of information does Bob receive?

$$\rho_A = \sum_x p(x)|x\rangle\langle x|$$

$$\rho_B = \sum_x p(x)\rho_x$$

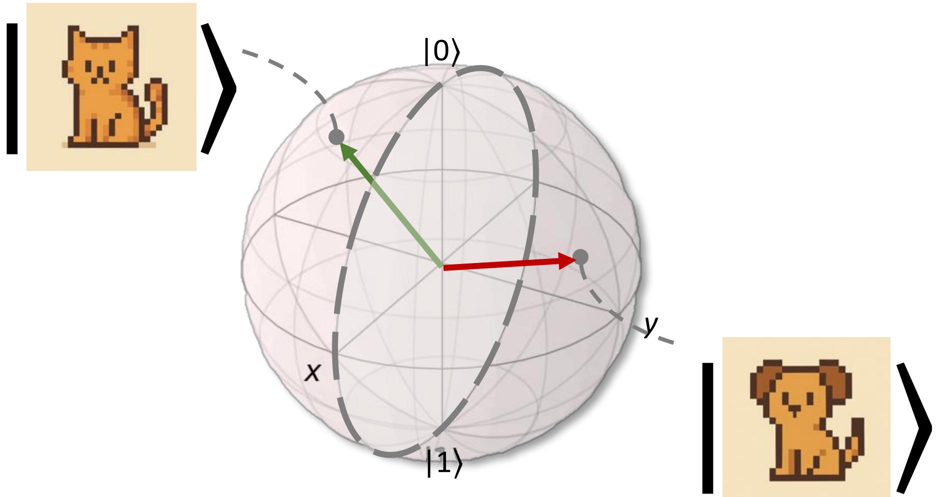
$$\rho_{AB} = \sum_x p(x)|x\rangle\langle x| \otimes \rho_x$$

- **Holevo's theorem** (formally)**:**
  - **Mutual information** $I(X:Z) \leq I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$
    - Where $S(\rho)$ is the von Neumann entropy.
  - Also common: $I(X:Z) \leq S(\rho_B) - \sum_x p(x)S(\rho_x)$
- **Superdense coding** (in Homework 3):
  - We can do better if Alice and Bob share pre-generated entanglement.
  - To communicate an m-bit message, Alice only needs to send $m/2$ qubits (given prior entanglement).

Classification

$|\psi_z\rangle \to z$ "Readout classical information from quantum states."

Example: Distinguishing quantum states



Metric for "overlap" or "similarity" of quantum states:

Fidelity $F(|\psi\rangle, |\phi\rangle) = |\langle\psi|\phi\rangle|^2$

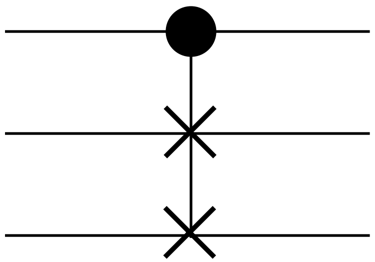
(Hilbert-Schmidt) inner product

How to estimate F for two **unknown** states?

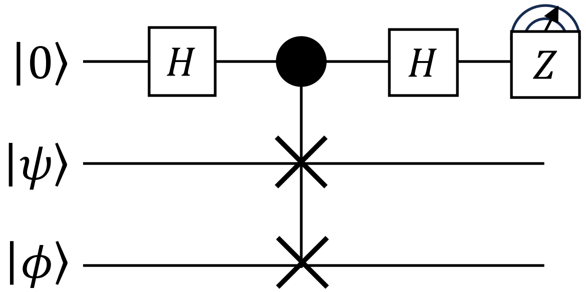
Simple Measurement Strategy

Testing fidelity between quantum states. Fidelity: $F = |\langle\psi|\phi\rangle|^2$

Controlled-SWAP (c-SWAP) gate:



Swap test:



Derive on board:

- What's the probability of measuring outcome $|0\rangle$?
- Performing **swap test** k times can estimate fidelity to an accuracy $\pm\sqrt{\frac{F(1-F)}{k}}$

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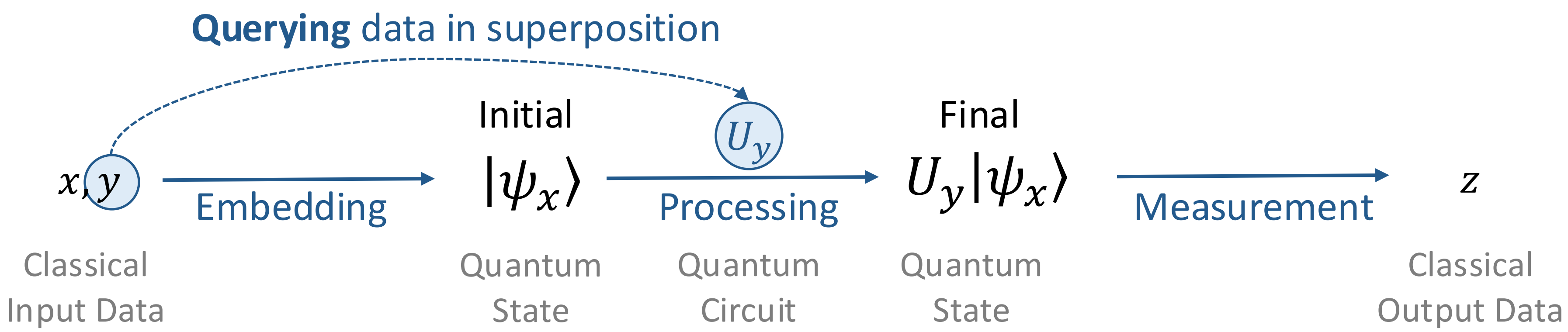
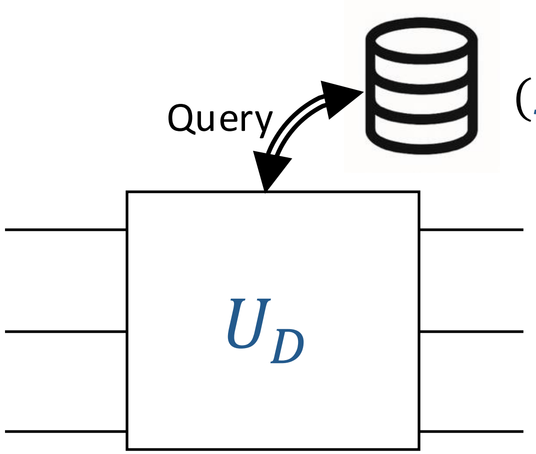


Quantum Oracles



$(x, y_x) \in D$: Dataset with (key, value) pairs

Initial

$$\sum_x \alpha_x |x\rangle = |\psi_x\rangle$$

Quantum State

Quantum Circuit

Final

$$|\psi_y\rangle = \sum_x \alpha_x |y_x\rangle$$

Quantum State

Example: $|x\rangle \rightarrow |f(x)\rangle$

$$\sum_x \alpha_x |x\rangle \rightarrow \sum_x \alpha_x |f(x)\rangle$$

"Evaluating **Boolean function** in superposition."

(key, value) = $\left(x, f(x)\right)$

Equivalent: Treating function f as RAM.

$$|i\rangle \rightarrow |m_i\rangle$$

$$\sum_i \alpha_i |i\rangle \rightarrow \sum_i \alpha_i |m_i\rangle$$

"Accessing **memory** in superposition."

(key, value) = (i, m_i)