

Lecture 1

Intro to QC - A CS perspective

Outline

- About the course
 - Why quantum computing?
 - What is quantum computing?
-

Plans for this course, (Four modules).

We will study:

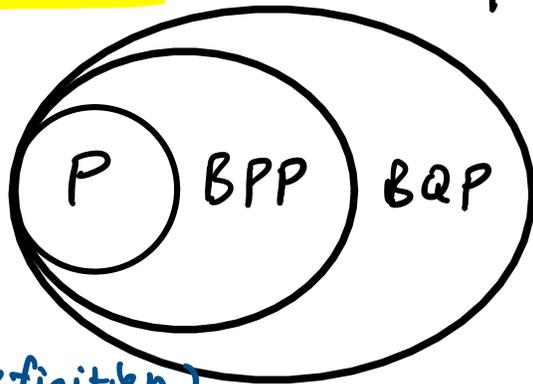
- ① The mathematical formulation of quantum mechanics in quantum computation.
- ② Practical issues of using (programming / running) quantum computers.
- ③ Some quantum algorithms and their complexity.
- ④ Basics of fault-tolerant quantum computation.

Q: Why do we study QC?

- A:
- ① Classical (Digital) computers have limitations.
 - ② Quantum computers can solve some computational problems exponentially faster*.
- (*: this is almost certainly true; hard to prove).

Limitations of Classical Computers:

Theoretical Limit (from complexity perspective)



(informal definition).

P: The set of problems that can be computed **efficiently**. ← In time that scales polynomially to the size of the input.

BPP: The set of problems with **efficient randomized** algorithms.

Extended Church-Turing Thesis.

The set of problems that can be computed in poly time is the same for any realistic models of computation.

(independent of how we build our computers)

or write our programs).

" A probabilistic Turing Machine can efficiently simulate any reasonable model of computation."

BQP: The set of problems with efficient quantum algorithm.

Remark: QC is a potential model that violates the extended Church-Turing Thesis.

Practical Limit (from engineering perspective)

- Slowdown of Moore's Law

transistor per chip doubles every two years
(at constant cost) ,

⇒ Not true since early 2000. (~2005).

- End of Dennard Scaling .

Power density of chip stays constant as transistors shrink.

⇒ No longer . "Hit the power wall."

dimension ↓ $\left\{ \begin{array}{l} \text{both voltage and current} \downarrow \text{ energy} \downarrow \\ \text{circuit delay} \downarrow \text{ op freq} \uparrow \end{array} \right\}$ power consumption same



"Quantum computer"

quantum sys B

uses poly(n) particles.

"If you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy." — Feynman.

What is a quantum computer?

Next few lectures will explain:

how QC stores and processes information differently than digital computer does.

For now, a sneak peek at the power of QC:

Information

	Classical Info	Quantum Info
Unit	bit	qubit
Possible state	0 or 1	<p>"Dirac notation"</p> $\alpha 0\rangle + \beta 1\rangle, \alpha, \beta \in \mathbb{C}$ $(\alpha ^2 + \beta ^2 = 1)$
Physical	transistor: on/off	<p>"Superposition"</p> Photons: polarization

Carrier

voltage in wire: high/low
coin: heads/tails
dice: faces

electron: spin up/down

Ion/Atom: energy levels

Superconductivity

circuits: current
invented at Yale.

Quantum bit vs. Random bit

(superposition vs. probability: a first look).

Communication Model:

Alice


send message
"Yes/No" →

Bob


Sending a random bit:

- Before sending, Alice knows her message. Bob doesn't.
(Bob is uncertain about Alice's bit.)
- Bob takes a guess: 50% "0" and 50% "1".
- After sending, Bob receives new info, e.g., bit is 0.

Probabilistic state represents the uncertainty about
the bit's value.
E.g., $\frac{1}{2}$ "0" + $\frac{1}{2}$ "1".

When we say we send a random bit:

Alice flips a fair coin, then she sends her bit
based on outcome.

The message being sent is in some **deterministic state**
we just don't know about it until we look at it.

Sending a quantum bit:

Quantum state represents the state of the qubit
that's being sent.

The qubit is literally in a **superposition state**.

E.g. $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$.

Nature "stores" the coefficients ($\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$) in the qubit
(e.g. photon).

Multiple bits.

Alice sends 3 Random bits:

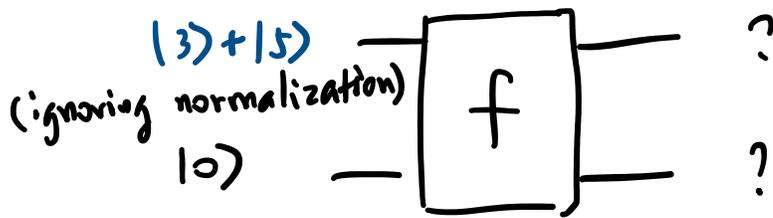
000	with prob. P_{000}	} She sends one of the 8 possible states using 3 registers.
001	with prob. P_{001}	
:	:	
111	with prob. P_{111}	

Alice sends 3 qubits:

$$\alpha_{000}|000\rangle + \alpha_{001}|001\rangle + \dots + \alpha_{111}|111\rangle$$

She sends a superposition state with 3 photons.
8 coefficients/amplitudes

we would get $|3\rangle$ with prob $(\frac{1}{\sqrt{2}})^2 = \frac{1}{2}$,
or $|5\rangle$ with prob $(\frac{1}{\sqrt{2}})^2 = \frac{1}{2}$.



$$(|3\rangle + |5\rangle)|0\rangle \longrightarrow (|3\rangle + |5\rangle) \cdot (|4\rangle + |6\rangle) \quad \text{☹️}$$

$$(|3\rangle + |5\rangle)|0\rangle \longrightarrow |3\rangle|4\rangle + |5\rangle|6\rangle \quad \text{😊}$$