

Lecture 7 CPSC 447/547 - Intro to QC

Quantum Teleportation

Now that we have finished the four principles of quantum computation, we can finally see its power.

Quantum teleportation is a great example of surprising things we can do with quantum circuits. Teleportation is what enables us to do quantum communication, to implement quantum architectures, and to perform universal fault-tolerant computation. It's literally everywhere. So what is it about?

Swap gate

To unentangled states : $|h_A\rangle \otimes |h_B\rangle \xrightarrow{\text{SWAP}} |h_B\rangle \otimes |h_A\rangle$

E.g. $|\psi_{AB}\rangle = \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} \otimes \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} \alpha_0\beta_0 \\ \alpha_0\beta_1 \\ \alpha_1\beta_0 \\ \alpha_1\beta_1 \end{bmatrix}$.

↓
- R - - - - - R - B - B - T

$$|\Psi'_{AB}\rangle = \begin{bmatrix} \alpha_0 \\ \beta_1 \end{bmatrix} \otimes \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} = \begin{bmatrix} \alpha_0 \alpha_0 \\ \alpha_0 \beta_1 \\ \beta_1 \alpha_0 \\ \beta_1 \beta_1 \end{bmatrix}$$

To entangled states: (Swap for each basis).

$$|\Psi_{AB}\rangle = \begin{bmatrix} \alpha_{00} \rightarrow |00\rangle \\ \alpha_{01} \rightarrow |01\rangle \\ \alpha_{10} \rightarrow |10\rangle \\ \alpha_{11} \rightarrow |11\rangle \end{bmatrix}$$

↓

Truth table

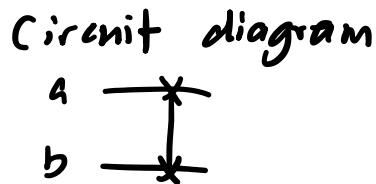
$$\begin{aligned} |00\rangle &\rightarrow |00\rangle \\ |01\rangle &\rightarrow |10\rangle \\ |10\rangle &\rightarrow |01\rangle \\ |11\rangle &\rightarrow |11\rangle \end{aligned}$$

$$|\Psi'_{AB}\rangle = \begin{bmatrix} \alpha_{00} \\ \alpha_{10} \\ \alpha_{01} \\ \alpha_{11} \end{bmatrix}$$

↓

Swap gate exchanges the amplitudes of $|01\rangle$ and $|10\rangle$.
("permutation")

$$\text{Swap} = \begin{bmatrix} 1 & & & \\ & 0 & 1 & \\ & 1 & 0 & \\ & & & 1 \end{bmatrix}$$



We have seen similar permutations before:

$\text{CNOT}_{a,b} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 0 & 1 \\ & & 1 & 0 \end{bmatrix}$

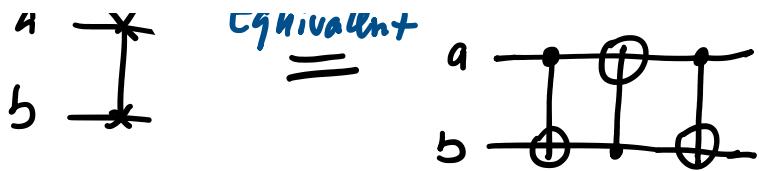
$\xrightarrow{\text{CNOT}_{a,b}}$ $\begin{bmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{bmatrix}$

$\text{CNOT}_{b,a} = \begin{bmatrix} 1 & & & \\ & 0 & 1 & \\ & 1 & 0 & \\ & & & 1 \end{bmatrix}$

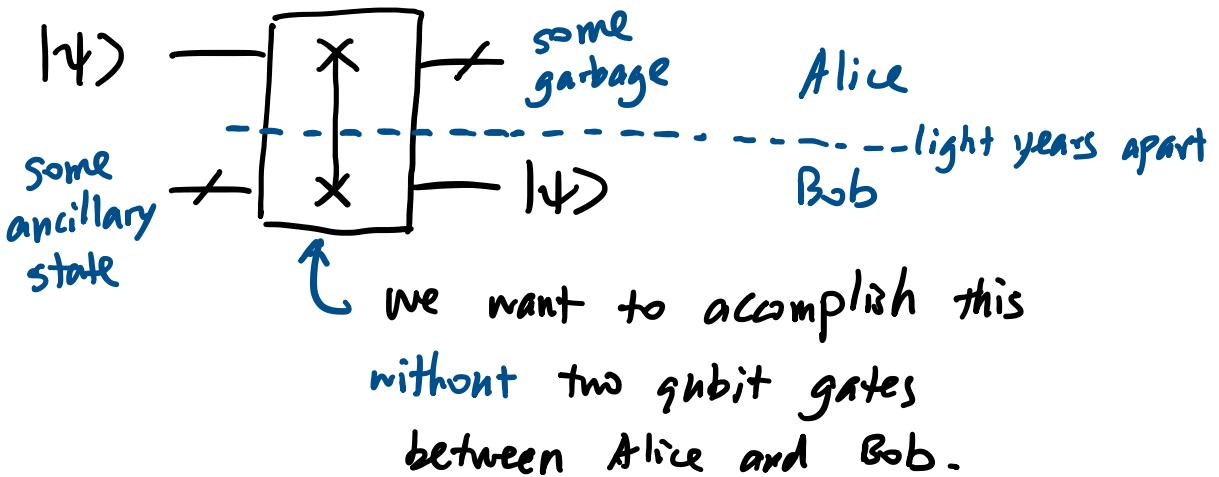
$\xrightarrow{\text{CNOT}_{b,a}}$ $\begin{bmatrix} \alpha_{00} \\ \alpha_{11} \\ \alpha_{10} \\ \alpha_{01} \end{bmatrix}$

Q: Can we use several CNOT gates to implement SWAP?

...



Teleportation (**Remote swap**).

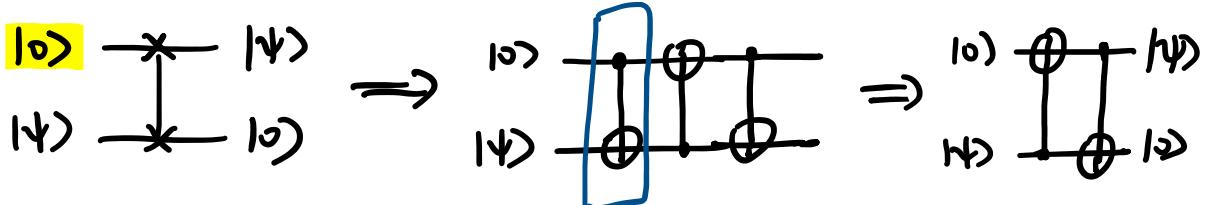


(Key : **Entanglement** between Alice and Bob !)

We are going to introduce **several tricks** in quantum circuits (one at a time) to help us invent the quantum teleportation circuit.

1. Simplified Swap.

Special case :



Redundant when control is |0> .

2. Conjugation relations.

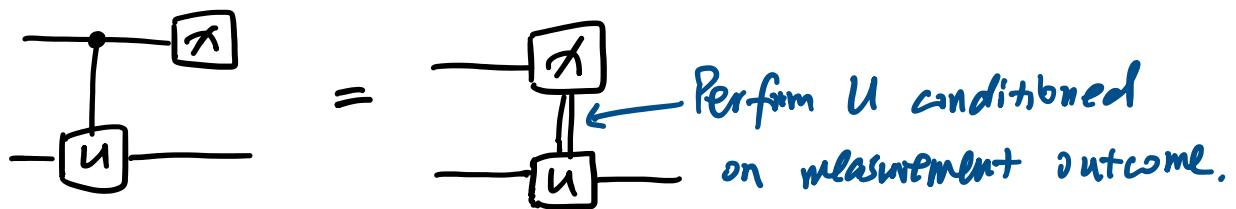
E.g. $X = HZH$, $HXH = Z$.

$$-\boxed{X}- = -\boxed{H} \boxed{Z} \boxed{H}-$$

$$-\boxed{H} \boxed{X}- = -\boxed{Z} \boxed{H}-$$

$$\begin{array}{c} \text{---} \\ \oplus \end{array} = \begin{array}{c} \text{---} \\ \oplus \\ \boxed{X} \end{array} = \begin{array}{c} \text{---} \\ \oplus \\ \boxed{H} \boxed{Z} \boxed{H} \end{array} = \begin{array}{c} \text{---} \\ \oplus \\ \boxed{H} \boxed{Z} \end{array}$$

3. Measurement and control.



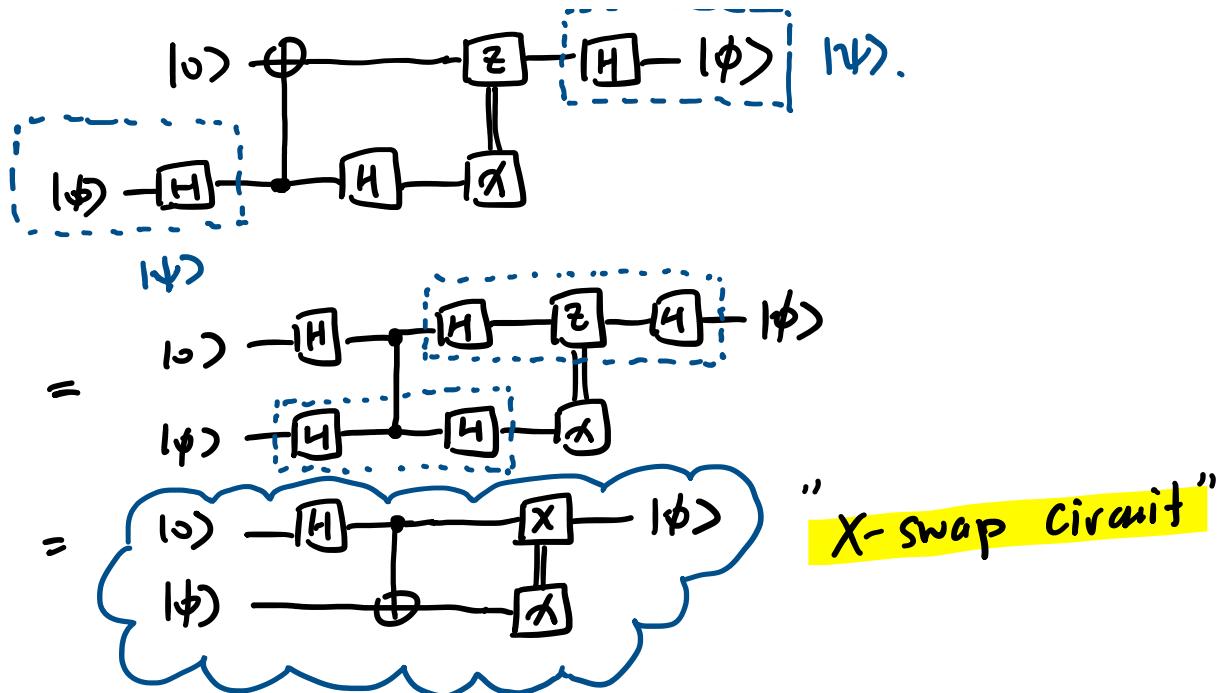
Now let's revisit the swap circuit:

$$\begin{array}{c} |0\rangle \\ |1\rangle \end{array} \begin{array}{c} \oplus \\ \text{---} \end{array} \begin{array}{c} |0\rangle \\ |1\rangle \end{array} = \begin{array}{c} |0\rangle \\ |1\rangle \end{array} \begin{array}{c} \oplus \\ \text{---} \\ \boxed{H} \end{array} \begin{array}{c} |0\rangle \\ |1\rangle \end{array}$$

$$= \begin{array}{c} |0\rangle \\ |1\rangle \end{array} \begin{array}{c} \oplus \\ \text{---} \\ \boxed{Z} \end{array} \begin{array}{c} |0\rangle \\ |1\rangle \end{array}$$

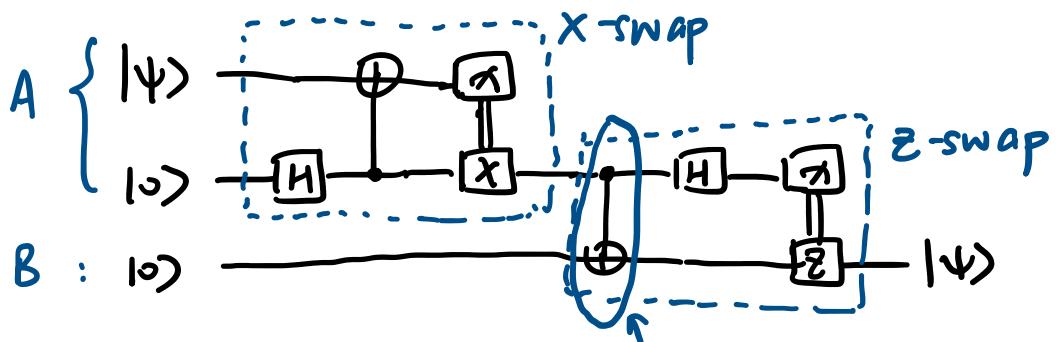
$$\begin{array}{c} \text{"Z-swap circuit"} \\ (\text{one-bit teleportation}) \end{array} = \begin{array}{c} |0\rangle \\ |1\rangle \end{array} \begin{array}{c} \oplus \\ \text{---} \\ \boxed{H} \end{array} \begin{array}{c} \boxed{Z} \\ \text{---} \\ \boxed{X} \end{array}$$

Similarly, we let $|+\rangle = H|\phi\rangle$.



(Two-bit) Teleportation Circuit.

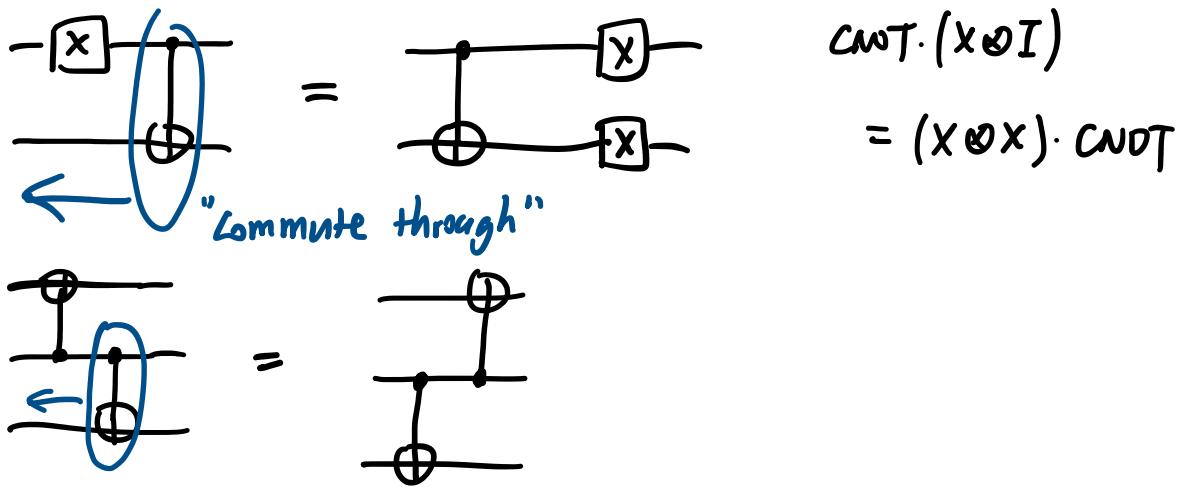
- "Alice sends $|ψ\rangle$ to Bob without two-qubit gates".
- The one-bit teleportation circuits do not work because of the CNOT gates.
- Let's try concatenate them :



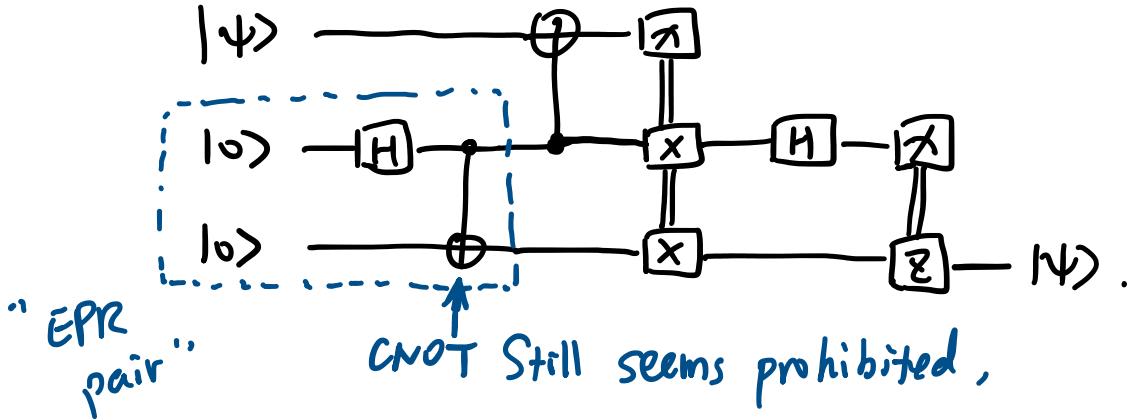
Problem: The only prohibited gate

Solution: let's try to "get rid of" the CNOT .

More conjugation relation tricks:



We will see (formal) **commutation/conjugation**
relations in next lecture.

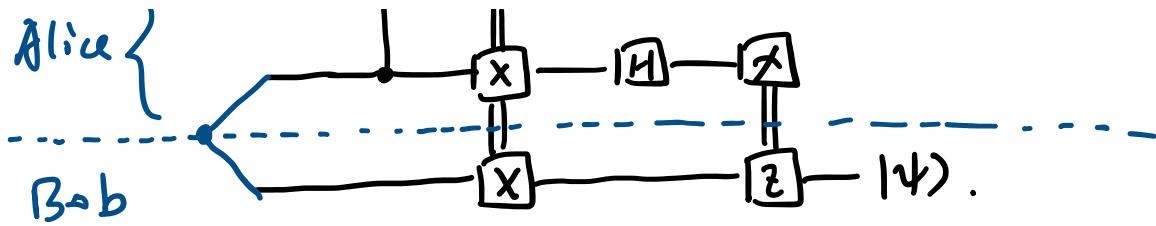


BUT, $|\psi\rangle$ does not depend on $|0\rangle$!

Alice and Bob can prepare EPR pair in advance.

Alice holds on to $|ψ\rangle$ and one of the qubits of the EPR pair, Bob holds another. ("Prior entanglement")

$$\langle \psi | - \oplus - |\chi \rangle$$



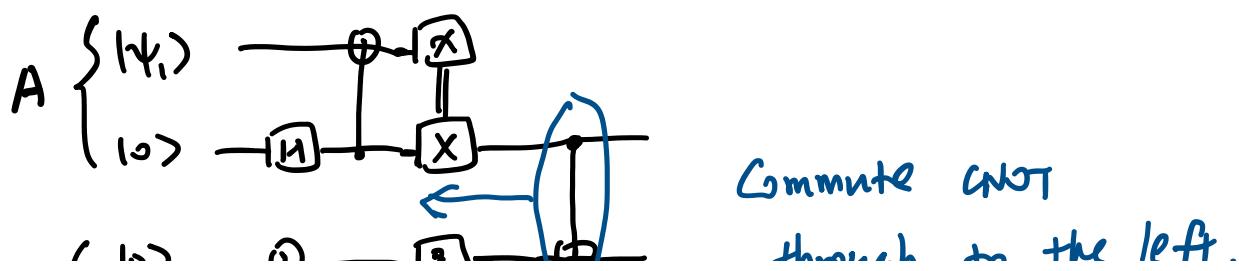
- No quantum gates between A and B during teleportation
- Classical communication still needed.
(No faster-than-light teleportation).

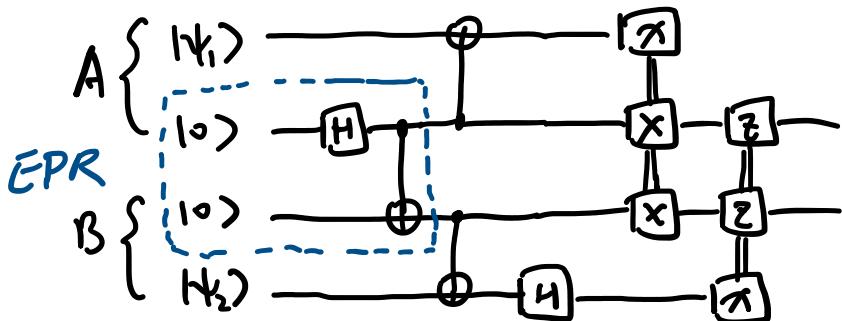
Application (beyond teleporting states)

Teleporting gates

Example : Remote CNOT

Alice has $|\psi_1\rangle$, Bob has $|\psi_2\rangle$, but they are light-years apart, can they perform $\text{CNOT}(|\psi_1\rangle, |\psi_2\rangle)$? (Given that they have shared entanglement in advance)





Alternative: Alice teleports her qubit to Bob,
 Bob performs CNOT locally,
 Bob teleports "her qubit" back to Alice.

(using one-bit teleportations, so also
 need one EPR pair.)