Lecture 25 CPSC 447/547 - Intro to QC

Q & A and Five-Qubit Code (optional)

Five-Qubit Code (Perfect Code)

$(n=5,\ k=1)$

Stabilizers:

$$S = \langle XZZXI,\ IXZZX,\ XIXZZ,\ ZXIXZ \rangle$$

Note ① $ZZXIX$ can be generated.

② S has four (independent) stabilizers

Codewords?

$(+1)$-eigenstate of every element in S.

$\Rightarrow$ For generators S_i of S and $|\psi\rangle$, a codeword can be found:

Projectors $\displaystyle\prod_{i=1}^{r} \frac{I+S_i}{2} |\psi\rangle \in C_S.$

$$= \Big(\frac{1}{2^r} \sum_{g\in S} g\Big) |\psi\rangle \quad \text{(up to normalization)}$$

every stabilizers (not only generators)
16 of them

Let's list **all stabilizers**:

$(XZ = -iY,\ ZX = iY)$

$S = \{ 11111, XZZX1, 1XZZX, X1XZZ, ZX1XZ, ZZX1X,$

$-XY1YX, -1ZYYZ, -YYZ1Z, -XXY1Y, -Z1ZYY, -YXXY1,$

$-1YXXY, -YZ1ZY, -ZYYZ1, -Y1YXX \}.$

Now we try $|\psi\rangle = |00000\rangle$.

$$|\psi_0\rangle = \frac{1}{4}\Big(|00000\rangle + |10010\rangle + |01001\rangle + |10100\rangle + |01010\rangle + |00101\rangle$$
$$- |11011\rangle - |00110\rangle - |11000\rangle - |11101\rangle - |00011\rangle - |11110\rangle$$
$$- |01111\rangle - |10001\rangle - |01100\rangle - |10111\rangle \Big)$$

We can also try $|\psi\rangle = |11111\rangle$.

$|\psi_1\rangle = \cdots$ and is orthogonal to $|\psi_0\rangle$! Basis $\{|0_L\rangle, |1_L\rangle\}$

Which one is $|0_L\rangle$ and which is $|1_L\rangle$?

That depends on our definition of Z_L.

(Because $Z_L|0_L\rangle = |0_L\rangle$, $Z_L|1_L\rangle = -|1_L\rangle$.)

Normalizers ?

What $h \in P_n$, s.t. $hg = gh$ for all $g \in S$?

$XXXXX$ and $ZZZZZ$

Anything else ? (Five-q code is dist 3, so $\exists$ weigh-3 operator in $N(S)$).

Note: if $h \in N(S)$, then $gh \in N(S)$ for $g \in S$.

E.g. $XXXXX \cdot XZZXI = IYYIX$

$XXXXX \cdot IXZZX = XIYYI$

$\vdots$

Btw, if $E = IYYIX$, cannot detect.

Let $X_L = XXXXX$, $Z_L = ZZZZZ$

then $|\psi_0\rangle = |0_L\rangle$, $|\psi_1\rangle = |1_L\rangle$.

Logical Hadamard?

$$H_L = HHHHH$$

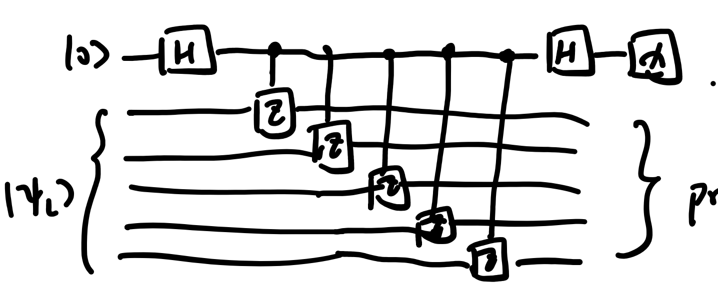
Because $X_L \overset{H_L}{\longleftrightarrow} Z_L$

Logical Clifford?

Logical measurement (in Z_L basis)?

$$|\psi_L\rangle = \alpha|0_L\rangle + \beta|1_L\rangle.$$

What's the circuit for obtaining $\begin{cases} |0_L\rangle \text{ with prob. } |\alpha|^2 \\ |1_L\rangle \text{ with prob. } |\beta|^2 \end{cases}$.



projected to $|0_L\rangle / |1_L\rangle$.