Analysis of reconstructed J/ψ meson candidates in $p\bar{p}$ collisions

Yongshan Ding

Mentor: Prof. James Russ
CMU Department of Physics
(Dated: August 24, 2014)

We present a RooFit analysis of inclusive J/ψ meson production in proton-antiproton collisions. We reconstruct $J/\psi \to \mu\mu$ decays with transverse momentum $p_T(J/\psi) > 6\text{GeV}/c$, in both of the invariant mass and decay lifetime dimensions. We determine the probability density function forms and perform the maximum likelihood fit of signal and background shapes to data distributions. Using the lifetime significance profiles, we distinguish the fraction of J/ψ prompt decay events from the decay of the long-lived b hadrons.

1. INTRODUCTION

In the experiment of proton-antiproton collision, J/ψ meson, the lowest charmonium state with quantum numbers $J^P = 1^-$, was produced from three major sources: directly produced J/ψ, prompt decays from heavier charmonium states, and decays of bottom-flavored hadrons (referred to as b hadrons or B in this report). The Tevatron measurement [1] of the J/ψ meson and b-hadron production cross sections was made over a large range of transverse momentum from zero to 20 GeV/c. And it allows us to reconstruct the J/ψ meson production and decay model from the decay channel $J/\psi \to \mu^+\mu^-$. The candidate events are selected through the process of the reconstruction of two oppositely charged muons. It is performed by recognizing and selecting qualified tracks measured in the tracking chambers of the CDF muon detectors [2]. Rigorous procedure of constraint fit is performed and iterated to find the optimal set of hits for muon identification. In this report, we demonstrate an analysis of the multi-dimensional fit of J/ψ production and decay in the $p\bar{p}$ collider. For both the invariant mass and decay lifetime distributions, the fits are examined in the three J/ψ p_T ranges of $6 < p_T(J/\psi) < 9$ GeV/c, $9 < p_T(J/\psi) < 15$ GeV/c, and $p_T(J/\psi) > 15$ GeV/c.

Although a substantial fraction of J/ψ meson are from the decays of b hadrons, we are able to separate the $B^+ \to J/\psi K^+$ events from the prompt charmonium production. The charmonium states immediately decay, while the b hadron has long lifetime characteristic. Due to the b hadron flight time, on the order of picoseconds[3], the J/ψ events from b hadron decays are displaced from that of the prompt productions. It allows us to measure the distance between the delayed J/ψ decay vertex and the beam line intersection.

2. MEASUREMENT AND IMPLEMENTATION

The reconstructed J/ψ invariant mass is calculated from the four-momenta of the two muons from the data selection and muon identification process[1],

$$m_{\mu\mu} = (P_1 + P_2)^2 = (m_\mu)^2 + 2E_1E_2 - 2P_1P_2, \quad (1)$$

The J/ψ candidates decay lifetime is then determined through the measurement of the J/ψ decay length on the transverse plane, L_{xy}. The proper time x is calculated using the relation:

$$x(J/\psi) = \frac{L_{xy}}{\beta\gamma} = L_{xy} \cdot \frac{m_{\mu\mu}}{p_T(J/\psi)} \quad (2)$$

2.1. Multi-Dimensional Modeling

The reconstructed $J/\psi \to \mu\mu$ data distribution has the input of two uncorrelated observables, namely the decay lifetime $x(J/\psi)$ and the invariant mass of di-muon pair $m_{\mu\mu}$. In order to formulate the 2-D model for the two uncorrelated variables, we determine to use the RooFit technique of p.d.f multiplication through `Class RooProdPdf`. Multiplication is the basic way to combine two or more p.d.f. models into higher dimensional model without correlations

$$\mathcal{F}(x, m_{\mu\mu}) = \mathcal{F}_T(x) \cdot \mathcal{F}_M(m_{\mu\mu}), \quad (3)$$

where $\mathcal{F}_T$ and $\mathcal{F}_M$ are the functional forms describing J/ψ decay lifetime distribution and the invariant mass distribution for the signal and background events, respectively.

The normalization of the function is taken care of automatically by RooFit, if the input functions are orthogonal and properly normalized p.d.f.s

$$\iint \mathcal{F}(x, m_{\mu\mu}) dt m_{\mu\mu} = \iint \mathcal{F}_T(x) dx \mathcal{F}_M(m_{\mu\mu}) dm_{\mu\mu}$$
$$= \int \mathcal{F}_T(x) dx \int \mathcal{F}_M(m_{\mu\mu}) dm_{\mu\mu} = 1 \quad (4)$$

Another benefit of using properly normalized p.d.f.s is that it allows us to simplify the projection calculation. The technique RooFit uses to calculate projection of each observable is integration. For example, the projection of J/ψ lifetime decay model $\mathcal{F}_T(x)$

$$\int \mathcal{F}(x, m_{\mu\mu}) dm_{\mu\mu} = \int \mathcal{F}_T(x) \mathcal{F}_M(m_{\mu\mu}) dm_{\mu\mu}$$
$$= \mathcal{F}_T(x) \int \mathcal{F}_M(m_{\mu\mu}) dm_{\mu\mu} = \mathcal{F}_T(x) \quad (5)$$

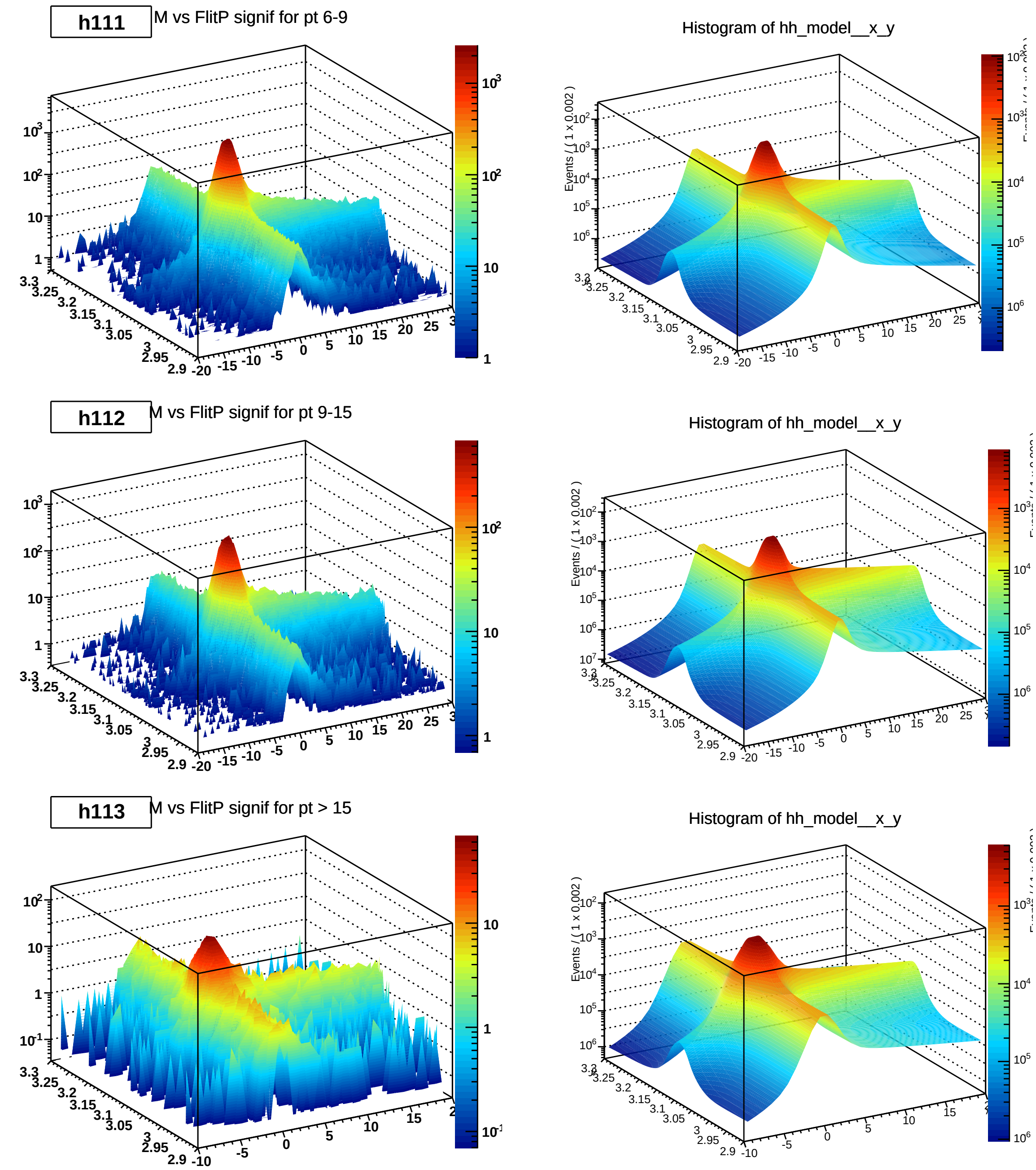

FIG. 1: (color online). Two dimensional fits to the reconstructed J/ψ meson production and decay in the ranges of $6 < p_T(\mu\mu) < 9$ GeV/c (top right panel), $9 < p_T(\mu\mu) < 15$ GeV/c (middle right panel), and $p_T(\mu\mu) > 15$ GeV/c (bottom right panel). The three left panels are the data histogram profiles for each p_T bin.

For all J/ψ p_T ranges, the likelihood function shapes for modeling the J/ψ signal distribution are determined and fitted based on the projection to each observable, which we will discuss in detail in the next few sections. The 2-D fits for all three sample p_T ranges are shown in Fig. 1

2.2. Lifetime Projection

The likelihood function form describing the decay lifetime shape is determined to be

$$\mathcal{F}_T(x) = n_p \times \mathcal{F}_P(x) + (1 - n_p) \times \mathcal{F}_B(x), \qquad (6)$$

where n_p is the faction of prompt J/ψ signal events, and $\mathcal{F}_P$ and $\mathcal{F}_B$ denote the functional forms describing prompt signal events and b-hadron decay events in the J/ψ lifetime distribution. The maximum likelihood fit is

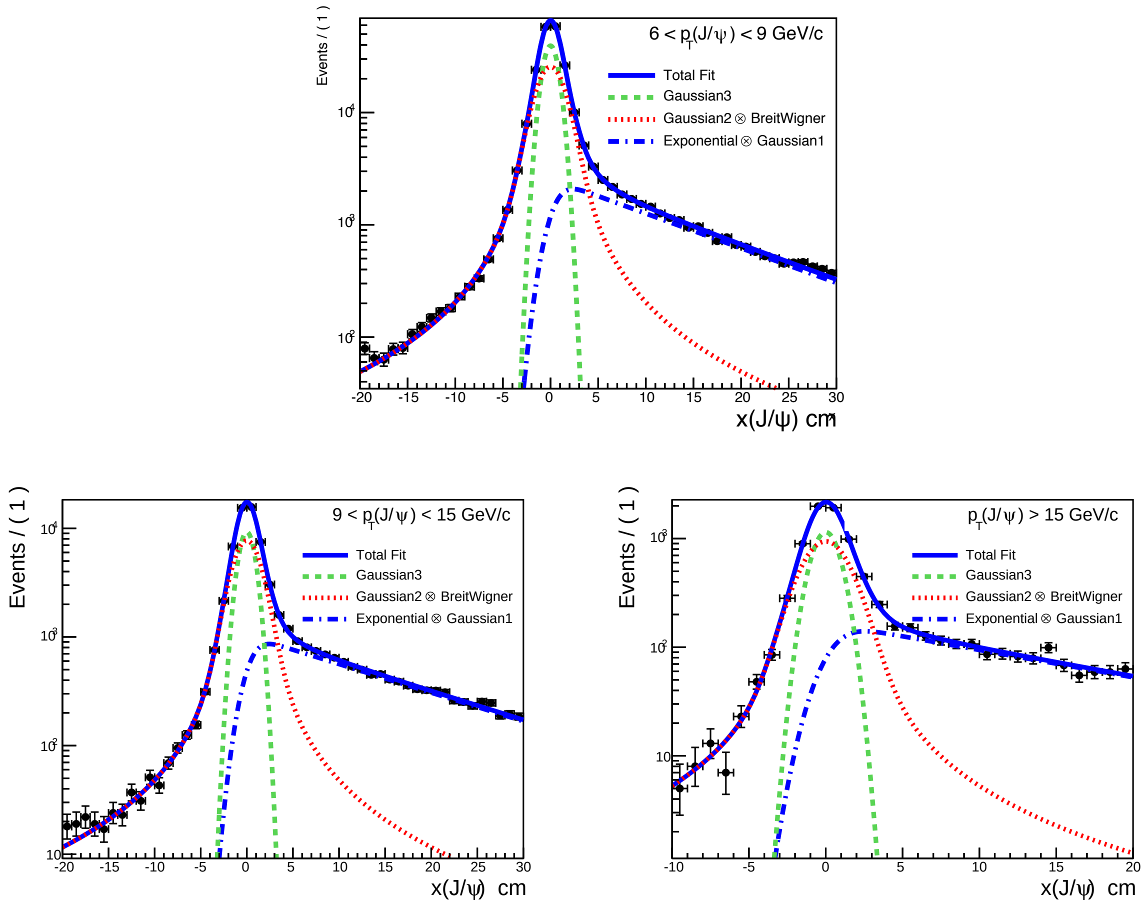

FIG. 2: (color online). Decay lifetime distribution of reconstructed $J/\psi \to \mu\mu$ events in the ranges of $6 < p_T(\mu\mu) < 9$ GeV/c (top panel), $9 < p_T(\mu\mu) < 15$ GeV/c (bottom left panel), and $p_T(\mu\mu) > 15$ GeV/c (bottom right panel). The horizontal axes, $x(J/\psi)$ denote the proper decay lifetime from Eq. (2). The points with error bars are data. The solid lines are the total likelihood fits to the J/ψ candidate events, the dashed lines are the fits to the Gaussian components of the model, the dotted lines are the fits to the signal events described by the convolution of the Gaussian and the Breit-Wigner function, and the dot-dashed lines are the fits to the background portion of the events, which is characterized as the convolution of the negative slope exponential distribution and the Gaussian distribution.

performed to determine the fit parameters, e.g. prompt signal fraction, n_p, from the data. The detail of the components of the likelihood function is explained in the following paragraphs.

2.2.1. Prompt Signal

The model for J/ψ decay lifetime prompt signal events is best described by the sum of a Gaussian distribution and a slightly more complicated signal function, labeled as G_3 and $\mathcal{F}_S(x)$ respectively:

$$\mathcal{F}_P(x) = n_g \times G_2(x, \sigma_2) + (1 - n_g) \times \mathcal{F}_S(x), \quad (7)$$

where n_g is the fraction of the Gaussian distribution, and Gaussian G_2 has mean 0 and width σ_2, on account of the measurement accuracy dependence on different events. The fit parameters are the fraction n_g and the width σ_2. For the additional signal function $\mathcal{F}_S$, another Gaussian distribution G_1 is convolved with a Breit-Wigner function BW as the following:

$$\mathcal{F}_S(x) = G_1(x, \sigma) \otimes BW(x, \Gamma), \quad (8)$$

where the Gaussian G_1 has mean 0 and width σ, the Breit-Wigner BW has mean 0 and width Γ, and $\otimes$ denotes a convolution. Both σ and Γ are parameters determined by the likelihood fit.

The (non-relativistic) Breit-Wigner formula, a.k.a the Cauchy distribution, is characterized as

$$BW(x - x_0, \Gamma) = \frac{1}{(x - x_0)^2 + \Gamma^2}, \quad (9)$$

where x_0 is the location of the peak, and Γ is the width of the peak. In this experiment, $x_0 = 0$, and Γ is allowed to float and is determined by the likelihood fit to data in each p_T ranges.The Breit-Wigner function is commonly convolved with a Gaussian function to describe a resonance shape due to the finite detector resolution. In our case, the signal sources include both the Gaussian G_2 and the convolution, which covers cases of non-Gaussian scattering or measurement effects.

Convolution integral is commonly used to describe experimental observables determined by both the signal response and the detector resolution. A convolution of two functions f and g is defined as

$$(f \otimes g)(x) = \int_{-\infty}^{+\infty} f(x')g(x-x')dx' \tag{10}$$

2.2.2. b-hadron Decay Signal

The $B \to J/\psi$ signal model is parametrized to be a convolution of a negative slope exponential distribution and a Gaussian distribution

$$\mathcal{F}_B(x) = \frac{1}{\tau}\exp(-\frac{x}{\tau})\theta(x) \otimes G_1(x,\sigma), \tag{11}$$

where τ is the mean of the exponential distribution, $\theta(x)$ is the step function defined as $\theta(x) = 1$ for $x \geq 0$ and $\theta(x) = 0$ for $x < 0$, and $\otimes$ denotes a convolution. The only two fit parameters here are exponential mean τ and gaussian width σ.

Notice that the Gaussian component $G_1(x,\sigma)$ in the $b-hadron$ decay model $\mathcal{F}_B$ is the same as the Gaussian $G_1(x,\sigma)$ in the convolution function $\mathcal{F}_S$. However, the Gaussian distribution $G_2(x,\sigma_2)$ in the model describing the prompt signal events, $\mathcal{F}_P$ has a width σ_2 that is allowed to vary independently.

2.3. Invariant Mass Projection

The di-muon invariant mass shape $\mathcal{F}_M$ has a less complicated functional parametrization. For the likelihood fit, it is determined to be

$$\begin{aligned}\mathcal{F}_M(m_{\mu\mu}) =& f_{Sig} \times \mathcal{M}_{Sig}(m_{\mu\mu}) \\ &+ (1-f_{Sig}) \times \mathcal{M}_{Bkg}(m_{\mu\mu}),\end{aligned} \tag{12}$$

where f_{Sig} is the fraction of reconstructed J/ψ invariant mass signal events, and $\mathcal{M}_{Sig}$ and $\mathcal{M}_{Bkg}$ are the fit models describing signal and background events in dimuon mass distribution. Similarly as for the J/ψ decay lifetime distribution, we perform maximum likelihood fit to determine the fit parameters. We now describe the function components in detail.

The di-muon mass signal model has to take the radiative energy loss into account. We introduce an asymmetric Crystal-Ball function to describe the signal events. Thus, the signal model consists of two components, the Crystal-Ball function $CB(m_{\mu\mu})$ and the Gaussian function $G_M(m_{\mu\mu})$. Thus the function is characterized as

$$\begin{aligned}\mathcal{M}_{Sig}(m_{\mu\mu}) =& f_{CB} \times CB(m_{\mu\mu} - m_0, \alpha, n, \sigma_{CB}) \\ &+ (1-f_{CB}) \times G_M(m_{\mu\mu} - m_0, \sigma_g),\end{aligned} \tag{13}$$

where the fit parameters are the fraction f_{CB} of the Crystal-Ball function events, the mean m_0 and width σ_g of the Gaussian distribution G_M, and the same mean m_0, the threshold α, the low-end tail power n, and gaussian core width σ_{CB} of the Crystal-Ball distribution CB. The Crystal-Ball function is given by

$$CB(m-m_0,\alpha,n,\sigma) = \begin{cases} \exp(-\frac{(m-m_0)^2}{2\sigma^2}) & : \frac{m-m_0}{\sigma} > -\alpha \\ \frac{(n/|\alpha|)^n \cdot \exp(-|\alpha|^2/2)}{(n/|\alpha| - |\alpha| - (m-m_0)/\sigma)^n} & : \frac{m-m_0}{\sigma} \leq -\alpha \end{cases}, \tag{14}$$

where m_0 is the position of the peak, α is the threshold dividing low-end power-law tail and the gaussian shape core, n is the power-law parameter, and σ is the width of the gaussian core. The Crystal-Ball function is best in describing an invariant mass with the effect of radiative energy loss.

The di-muon mass background is simply modeled as a first-order polynomial. The order of background polynomial depends on each J/ψ transverse momentum bin, and a linear background is used to describe events with $p_T \geq 2.25$ GeV/c [1]. Therefore, for our purposes, the background polynomial shape is parametrized as

$$\mathcal{M}_{Bkg}(m_{\mu\mu}) = b + k \times m_{\mu\mu}, \tag{15}$$

where the only fit parameter is the slope of the linear background distribution labeled k. The constant term b is determined by the normalization of the function.

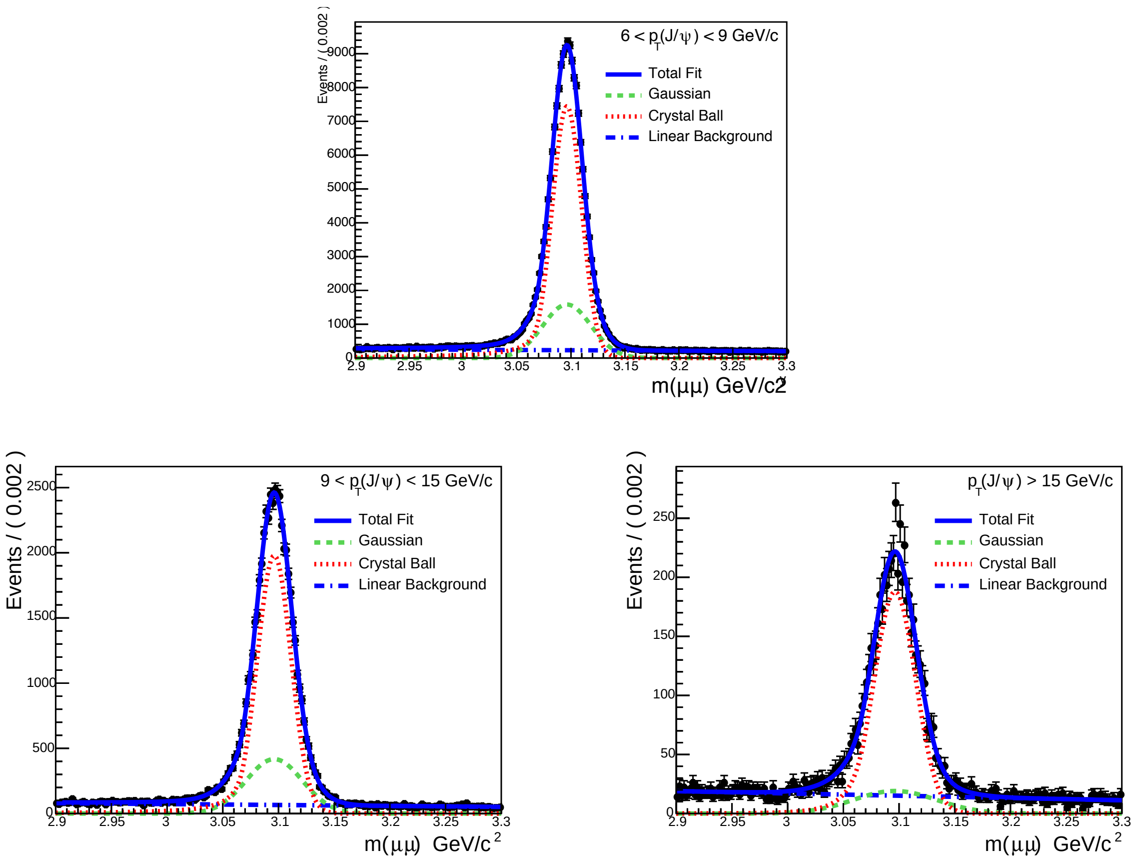

FIG. 3: (color online). Fits to invariant mass distribution of reconstructed $J/\psi \to \mu\mu$ events in the ranges of $6 < p_T(\mu\mu) < 9$ GeV/c (top panel), $9 < p_T(\mu\mu) < 15$ GeV/c (bottom left panel), and $p_T(\mu\mu) > 15$ GeV/c (bottom right panel). The horizontal axes, $m(\mu\mu)$, denote the invariant mass from Eq. (1).The points with error bars are data. The solid lines are the total likelihood fits to the J/ψ candidate events, the dashed lines are the fits to the Gaussian components of the mass shape model, the dotted lines are the fits to the portion of events modeled by the Crystal-Ball function, and the dot-dashed lines are the fits to the linear background events.

3. THE FITS AND ASSESSMENT

The fits to the reconstructed J/ψ meson decay lifetime and di-muon mass in each sample p_T bin are shown in Fig. 2 and Fig. 3. Obtained from the likelihood procedure, the values of the fit parameters we used to construct the multi-dimensional model in all J/ψ transverse momenta ranges are shown in Table I and Table II. The fit parameter n_p is then used to determine the fraction of J/ψ events from the decays of b-hadrons, n_b, using the relation:

$$n_b = 1 - n_p \tag{16}$$

Thus, we find the result in each transverse momentum bin to be 0.141 ± 0.001 (at $6 < p_T(\mu\mu) < 9$ GeV/c), 0.216 ± 0.002 (at $9 < p_T(\mu\mu) < 15$ GeV/c), and 0.235 ± 0.007 (at $p_T(\mu\mu) > 15$ GeV/c).

For all transverse momenta ranges, the binned data distribution are compared to the fit shapes determined by the likelihood procedure. We have also performed the pull distribution tests to assess the likelihood fitting accuracy. The pull, a.k.a. the normalized residual, for each bin content, is defined as the difference between the fit projection and the data divided by the statistical error on the histogram.

The model we constructed characterizes two independent observables, the decay proper lifetime x and the invariant mass $m_{\mu\mu}$. The J/ψ mass sidebands are properly checked to verify this correlation.

TABLE I: Summary of the likelihood fits to the reconstructed J/ψ meson decay lifetime shapes. The values of the fit parameter and the statistical uncertainty are determined by the fits for all transverse momenta ranges.

Parameter Name	$(6 < p_T(\mu\mu) < 9$ GeV/$c)$ Value with Errors	$(9 < p_T(\mu\mu) < 15$ GeV/$c)$ Value with Errors	$(p_T(\mu\mu) > 15$ GeV/$c)$ Value with Errors
n_g	0.43 $\pm$ 0.03	0.39 $\pm$ 0.07	0.40 $\pm$ 0.21
n_p	0.859 $\pm$ 0.001	0.784 $\pm$ 0.002	0.765 $\pm$ 0.007
σ	1.29 $\pm$ 0.03	1.27 $\pm$ 0.05	1.34 $\pm$ 0.18
σ_2	0.84 $\pm$ 0.02	0.87 $\pm$ 0.04	0.90 $\pm$ 0.13
τ	14.07 $\pm$ 0.20	16.49 $\pm$ 0.38	17.26 $\pm$ 1.69
Γ	1.09 $\pm$ 0.04	0.90 $\pm$ 0.09	0.80 $\pm$ 0.22

TABLE II: Summary of the likelihood fits to the reconstructed J/ψ meson invariant mass shapes. The values of the fit parameter and the statistical uncertainty are determined by the fits for all transverse momenta ranges.

Parameter Name	$(6 < p_T(\mu\mu) < 9$ GeV/$c)$ Value with Errors	$(9 < p_T(\mu\mu) < 15$ GeV/$c)$ Value with Errors	$(p_T(\mu\mu) > 15$ GeV/$c)$ Value with Errors
α	1.89 $\pm$ 0.02	1.83 $\pm$ 0.05	1.52 $\pm$ 0.14
σ_{CB}	0.0135 $\pm$ 0.0001	0.0151 $\pm$ 0.0003	0.0187 $\pm$ 0.0008
f_{CB}	0.75 $\pm$ 0.02	0.75 $\pm$ 0.05	0.84 $\pm$ 0.07
f_{Sig}	0.795 $\pm$ 0.003	0.801 $\pm$ 0.005	0.643 $\pm$ 0.009
σ_g	0.0223 $\pm$ 0.0005	0.0253 $\pm$ 0.0012	0.0374 $\pm$ 0.0073
m_0	3.09628 $\pm$ 0.00004	3.09618 $\pm$ 0.00009	3.09633 $\pm$ 0.00040
n	1.2 $\pm$ 0.1	1.6 $\pm$ 0.3	6.0 $\pm$ 4.8
k	-0.210 $\pm$ 0.011	-0.249 $\pm$ 0.008	-0.257 $\pm$ 0.007

4. SUMMARY

We have constructed the likelihood models for the J/ψ mesons produced in the $p\bar{p}$ interactions. We have performed the two-dimensional maximum likelihood fit to the data distribution and we have tested the accuracy of the fit shapes by the pull distributions. Using the displaced decay from long lived b-hadrons, we have separated its events from that of at the primary vertices. We find the $B \to J/\psi$ fraction of the reconstructed dimuon events to be 0.141 ± 0.001 (at $6 < p_T(\mu\mu) < 9$ GeV/c), 0.216 ± 0.002 (at $9 < p_T(\mu\mu) < 15$ GeV/c), and 0.235 ± 0.007 (at $p_T(\mu\mu) > 15$ GeV/c).

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Acknowledgments

Gratefully acknowledges Dr. James Russ for his valuable guidance and supervision over this project.